Rossmoyne Senior High School

Semester Two Examination, 2019

Question/Answer booklet

MATHEMATICS APPLICATIONS UNITS 3 AND 4
Section Two:
Calculator-assumed

SOLUTIONS

Student number: In figures

In words   _____

Your name   _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1.   The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2.   Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3.   You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4.   Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5.   It is recommended that you do not use pencil, except in diagrams.

6.   Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7.   The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.
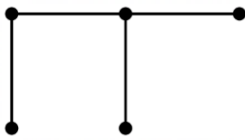
Working time: 100 minutes.

Question 9 **(6 marks)**

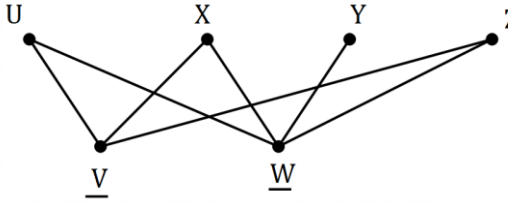(a) Connected planar graph G_1 has 29 faces and 42 edges. Determine the number of vertices G_1 has. **(2 marks)**

Solution
$29 + v - 42 = 2$
$v = 15$
Specific behaviours
✓ indicates use of Euler's formula with numbers
✓ correct number

(b) Draw graph G_2 so that it has 5 vertices, is a tree and is not semi-Hamiltonian. **(2 marks)**

Solution
(example solution)
Specific behaviours
✓ tree using 5 vertices
✓ not semi-Hamiltonian

(c) Graph G_3 is shown below. Redraw G_3 to clearly show that it is bipartite. **(2 marks)**
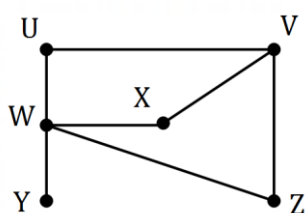
Solution
(example solution)
Specific behaviours
✓ clearly bipartite
✓ correct bipartite graph

Question 10   **(11 marks)**

The table below show the length (L) and the weight (W) of some mackerel, a species of fish.

L (cm)	25	32	39	30	42	28	35	44	33	36	27	38
W (kg)	0.21	0.39	0.77	0.41	0.83	0.38	0.55	0.85	0.52	0.62	0.25	0.58

(a) By viewing a scatterplot of this data on your calculator, identify and describe the direction and form of the relationship between the variables.   (2 marks)

Solution
Direction is positive and form is linear.
Specific behaviours
✓ states positive direction
✓ states linear form

(b) Calculate the correlation coefficient between the variables and use it to describe the strength of their association.   (2 marks)

Solution
$r = 0.975$.
Hence the strength of association is strong.
Specific behaviours
✓ correct coefficient
✓ states strong association

(c) What percentage of the variation in weight can be explained by the variation in the length for this sample?   (1 mark)

Solution
$r^2 = 0.950 \Rightarrow 95\%$
Specific behaviours
✓ correct percentage

(d) Using length as the explanatory variable, determine the equation of the least-squares line to model the linear relationship between the variables.   (2 marks)

Solution
$W = 0.0345L - 0.647$
Specific behaviours
✓ correct coefficients
✓ uses correct variables

(e) What increase in weight can be expected for each additional centimetre in length for these mackerel?   (1 mark)

Solution
0.0345 kg or 34.5 grams
Specific behaviours
✓ weight with units

(f) Another mackerel in the sample had a length of 41 cm. Predict the weight of this fish and comment on two factors that support the reliability of this prediction.   (3 marks)

Solution
$\widehat{W}(41) = $ 0.77 kg. Reliable because a strong linear association exists, and prediction is interpolated.
Specific behaviours
✓ correct prediction
✓ notes interpolation
✓ notes strong association

Question 11 **(6 marks)**

The number of tickets T_n remaining for a musical event at the start of day n can be modelled by the recursive rule $T_{n+1} = T_n - 17$. At the start of day 1, there were 646 tickets remaining.

(a) Complete the table below. (2 marks)

n	5	10	15	20
T_n	**578**	**493**	**408**	**323**

Solution
See table
Specific behaviours
✓ at least two correct entries
✓ all correct entries

(b) Would the graph of T_n against n appear to be linear or non-linear? (1 mark)

Solution
Linear
Specific behaviours
✓ correct type

(c) The nth term rule for this sequence is $T_n = b + an$. Determine the value of the constant a and the value of the constant b. (2 marks)

Solution
$T_n = 646 + (n-1)(-17)$
$\quad = 646 - 17n + 17$
$\quad = 663 - 17n$
Hence $a = -17$ and $b = 663$.
Specific behaviours
✓ value of a
✓ value of b

(d) At the start of day k, fewer than 50 tickets remain. State the value of k. (1 mark)

Solution
$k = 37$
Specific behaviours
✓ correct value

Question 12   **(6 marks)**

An investor is considering placing the sum of $96 000 into one of three investments for 2 years.

Investment A offers a return of 6.82% compounded daily.

(a)   Determine the profit that the investor would achieve through investment A.   (2 marks)

Compound Interest	
N	730
I%	6.82
PV	−96000
PMT	0
FV	110028.0628
P/Y	365
C/Y	365

Solution
$FV = \$110\,028.06$
Profit $= \$14\,028.06$
Specific behaviours
✓ future value
✓ profit nearest cent

Investment B offers a return of 6.83% compounded monthly.

(b)   Calculate the difference in profit that the investor would achieve through investment B compared to investment A.   (2 marks)

Compound Interest	
N	24
I%	6.83
PV	−96000
PMT	0
FV	110008.8613
P/Y	12
C/Y	12

Solution
$FV = \$110\,008.86$
Difference $= 28.06 - 8.86 = \$19.20$
Specific behaviours
✓ future value for B
✓ difference to nearest cent

Investment C offers a return of 6.87% compounded quarterly.

(c)   Rank the three investments in order, from highest to lowest profit. Justify your answer.

                                                (2 marks)

Investment C

$110011.13

Solution
Effective interest rates are:
$A: 7.0573\%$
$B: 7.0479\%$
$C: 7.0490\%$
Ranking: A, C, B
Specific behaviours
✓ effective rates or other comparison %
✓ correct ranking (must be justified)

See next page

Question 13 **(5 marks)**

A young person has a bank account that pays no interest. At the start of each week they withdraw a fixed percentage of the balance to spend and then deposit a lump sum from their wages. The balance of the account at the end of week n is given by A_n, where

$$A_{n+1} = 0.92A_n + 28, \qquad A_1 = 74.92.$$

(a) Calculate A_5 and explain what this figure represents.

(2 marks)

Solution
$A_5 = \$153.93$
This is the amount in the account at the end of week 5.
Specific behaviours
✓ amount to nearest cent
✓ explanation

(b) State what percentage of the balance they withdraw each week. (1 mark)

Solution
$1 - 0.92 = 0.08 = 8\%$
Specific behaviours
✓ correct percentage

(c) The young person plans to continue operating the account in this manner until it holds at least $500. Comment on this plan. (2 marks)

Solution
Not possible for the balance of this account to exceed $350 (long term steady state) and so plan is not realistic.
Specific behaviours
✓ states maximum account balance
✓ makes sensible comment

Question 14 **(6 marks)**

A random sample of 450 TAFE students studying the same diploma was taken. Their study mode and enjoyment of the course is summarised in the table below.

		Study mode	
		Attend campus	Online
Level of course enjoyment	High	114	45
	Moderate	140	52
	Low	55	23
	None	16	5

(a) Show that the study mode of approximately 72% of the students in the sample is to attend campus. (2 marks)

Solution
$114 + 140 + 55 + 16 = 325$
$325 \div 450 = 0.722 \approx 72\%$
Specific behaviours
✓ column total
✓ correct value as percentage

(b) Complete the two-way frequency table below to show the column percentages for the above data, rounding to the nearest whole number. (2 marks)

		Study mode	
		Attend campus	Online
Level of course enjoyment	High	35	36
	Moderate	43	42
	Low	17	18
	None	5	4

Solution
See table (no penalty if not rounded)
Specific behaviours
✓ all values in one column
✓ both columns correct and whole numbers used

(c) Explain whether the data suggests the presence of an association between the variables. (2 marks)

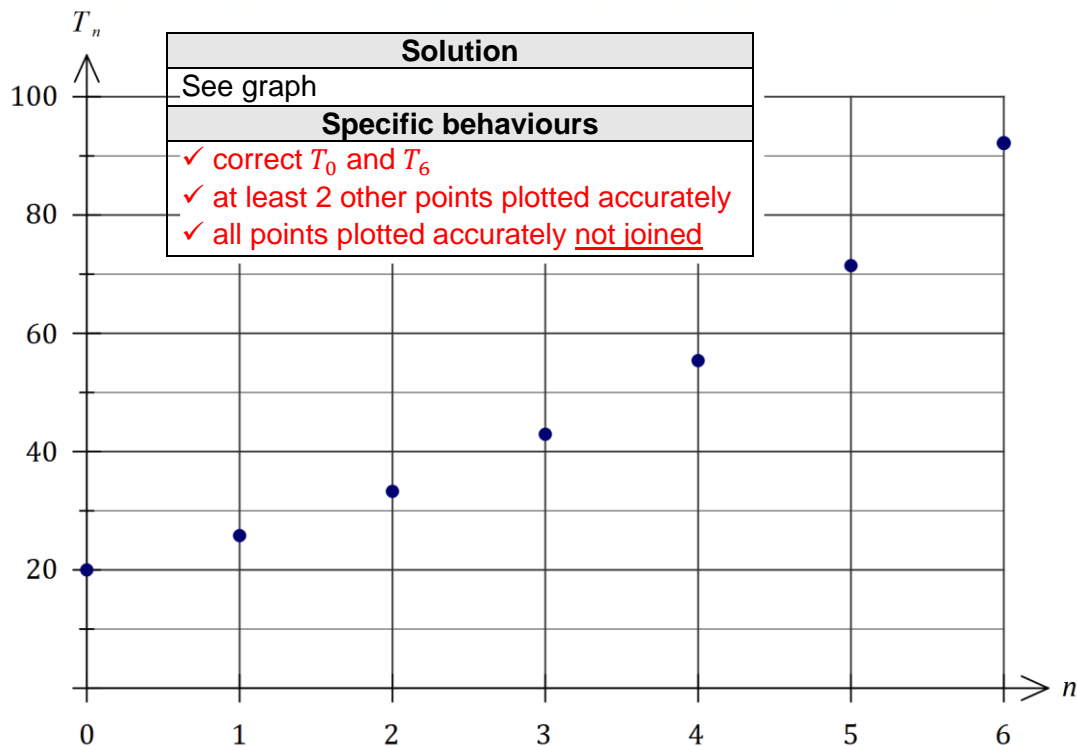| Solution |
|---|
| No evidence that level of course enjoyment for this diploma is associated with study mode because across all four rows, both values are very similar. |
| **Specific behaviours** |
| ✓ states no association with an explanation |
| ✓ explanation for no association |

Question 15 (6 marks)

An electric kiln is switched on and after n minutes, the temperature of the kiln T_n is given by the recursive rule $T_{n+1} = 1.29T_n$. The initial temperature of the kiln, T_0, is $20°C$.

(a) Calculate the temperature of the kiln after 1 minute. (1 mark)

Solution
$T_1 = 1.29 \times 20 = 25.8°C$
Specific behaviours
✓ correct temperature

(b) Graph the temperature of the kiln for $n = 0, 1, 2, 3, 4, 5$ and 6 on the axes below. (3 marks)

Solution
See graph
Specific behaviours
✓ correct T_0 and T_6
✓ at least 2 other points plotted accurately
✓ all points plotted accurately <u>not joined</u>

(c) Name the type of growth displayed in the graph. (1 mark)

Solution
Exponential
Specific behaviours
✓ correct term

(d) During which minute does the temperature of the kiln first increase by more than $100°C$? (1 mark)

Solution
During the 13th minute (from $425°C$ to $707°C$)
Specific behaviours
✓ correct minute

Question 16   **(8 marks)**

A customer in a store is offered a reducible interest loan that attracts interest of 8.4% per annum compounded monthly to purchase a \$2 545.39 computer and accessories. The monthly loan repayment is \$151.

The customer set up the spreadsheet below to analyse the loan, rounding the displayed figures to the nearest cent.

Month	Balance at start of month (\$)	Interest for month	Repayment	Balance at end of month (\$)
1	2 545.39	17.82	151.00	2 412.21
2	2 412.21	16.89	151.00	2 278.09
3	**2 278. 09**	**15. 95**	**151. 00**	**2143. 04**

(a)   Complete row 3 of the spreadsheet.   (3 marks)

Solution
See table
Specific behaviours
✓ opening balance and repayment
✓ interest
✓ closing balance all to nearest cent

(b)   Determine a simplified recurrence relation for B_n, the loan balance at the start of month n.
         (2 marks)

Solution
$1 + 8.4 \div 12 \div 100 = 1.007$
$B_{n+1} = 1.007B_n - 151, \qquad B_1 = 2545.39$
Specific behaviours
✓ interest multiplier
✓ recurrence relation and first term using B_n

(c)   State the number of repayments required to pay off the loan.   (1 mark)

Compound Interest

N	18.00002128
I%	8.4
PV	2545.39
PMT	−151
FV	0
P/Y	12
C/Y	12

1

Solution
18 repayments
Specific behaviours
✓ correct number

(d)   Determine the total amount of interest paid over the life of the loan.   (2 marks)

Amortization

PM1	1
PM2	18
I%	8.4
PV	2545.39
PMT	−151
P/Y	12
C/Y	12
BAL	
INT	
PRN	
ΣINT	−172.6132016
ΣPRN	

Solution
Total payments: $18 \times 151 = 2718$
$I = 2718 - 2545.39 = \$172.61$
Specific behaviours
✓ total payments
✓ interest to nearest cent

See next page   SN085-146-4

Question 17 **(8 marks)**

A delivery driver must leave depot D at $1\!:\!30$ pm, deliver packages to three schools (X, Y and Z) and then return to the depot. The table below shows the travel times between the various locations in minutes.

	X	Y	Z
D	49	48	51
Z	54	50	
Y	52		

(a) Construct a weighted graph to represent this information. (3 marks)

Solution

(example solution)

Specific behaviours

✓ four labelled vertices
✓ six edges
✓ correct weights

(b) Explain why the graph in part (a) is a complete graph. (2 marks)

Solution

It is a simple graph in which every vertex is joined to every other vertex by an edge.

Specific behaviours

✓ states graph is simple
✓ states all pairs of edges joined

(c) Determine the route the driver should take to minimise delivery time and calculate the time they will arrive back at the depot if they spend 12 minutes at each school handing over the packages. (3 marks)

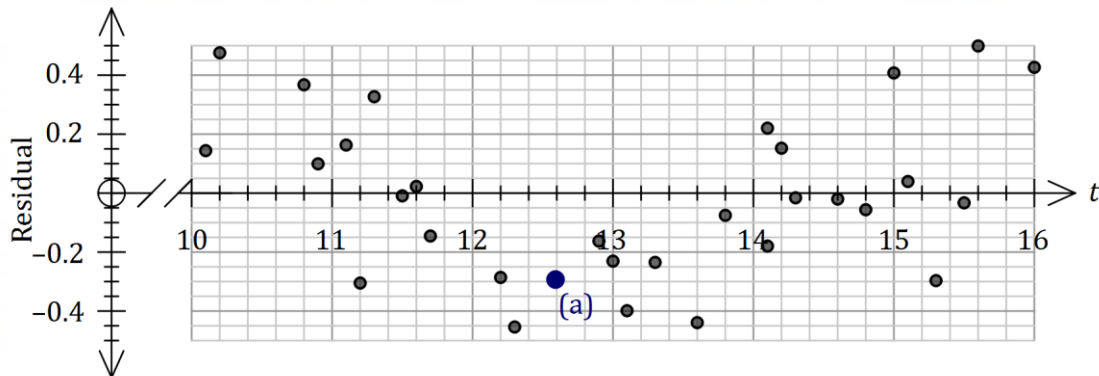| **Solution** |
|---|

$DXYZD = 202$
$DXZYD = 201$
$DXZYD = 205$

Route for minimum time is $DXZYD$ (or reverse)

Time taken is $201 + 3 \times 12 = 237$ minutes.

Arrive back at $5\!:\!27$ pm

Specific behaviours

✓ evidence of times for at least 2 routes
✓ lists optimum route
✓ correct arrival time with pm

Question 18   **(9 marks)**

An experiment involved measuring the voltage v in a circuit after t seconds. The equation of the least-squares line to model the linear relationship between v and t is given by $v = 5.77 - 0.32t$ and $r_{vt} = -0.945$.

The residual plot for the linear model is shown below.

(a)   After 12.6 seconds, the measured voltage was 1.44. Calculate the residual for this point and add it to the plot above.   (3 marks)

Solution
$v = 5.77 - 0.32(12.6) = 1.738$
Residual $= 1.44 - 1.738 = -0.298$
Specific behaviours
✓ predicts voltage
✓ calculates residual
✓ plots residual (FT)

(b)   Explain what information in this question

       (i)   supports the use of the linear model.   (2 marks)

Solution
The correlation coefficient of -0.945 indicates a strong association between the variables.
Specific behaviours
✓ states correlation
✓ states that correlation is strong

       (ii)   does not support the use of the linear model.   (2 marks)

Solution
A pattern is evident in the residuals (tend to change from positive to negative to positive as time increases) which indicates that the linear model is not appropriate.
Specific behaviours
✓ refers to residual plot
✓ states a pattern is evident

(c)   The residual for the voltage measured after 15 seconds is shown on the plot above. Determine what voltage was measured at this time.   (2 marks)

Solution
$5.77 - 0.32(15) = 0.97$
$v - 0.97 = 0.41 \Rightarrow v = 1.38$
Specific behaviours
✓ residual from plot $[0.40, 0.42]$
✓ measured voltage $[1.37, 1.39]$

See next page

Question 19 (7 marks)

An annuity compounds interest annually and its value after n withdrawals can be modelled using the recurrence relation

$$T_{n+1} = 1.071T_n - 38\,500, \qquad T_0 = 435\,000.$$

(a) Use the relation to state

(i) the annual percentage interest rate. (1 mark)

Solution
7.1%
Specific behaviours
✓ correct value as percentage

(ii) the initial value of the annuity. (1 mark)

Solution
$435 000
Specific behaviours
✓ correct value

(b) Calculate the balance of the annuity, to the nearest dollar, after 2 withdrawals. (1 mark)

Compound Interest

N	20
I%	7.1
PV	435000
PMT	-38500
FV	-119389.266
P/Y	1
C/Y	1

Solution
$T_2 = \$419\,229$
Specific behaviours
✓ correct value to nearest dollar

(c) The annuity is closed after 20 withdrawals. Calculate the total interest paid by the annuity up to this time. (2 marks)

Amortization

PM1	1
PM2	20
I%	7.1
PV	435000
PMT	-38500
P/Y	1
C/Y	1
BAL	
INT	
PRN	
ΣINT	-454389.266

Solution
$T_{20} = 119\,389.27$
$T_{20} - (T_0 - 20 \times 38500) = \$454\,389.27$
Specific behaviours
✓ indicates correct method
✓ correct amount to the nearest cent

(d) From the outset, the annual withdrawal can be reduced so that the annuity becomes a perpetuity. Briefly explain what a perpetuity is and determine the withdrawal required. (2 marks)

Solution
A perpetuity is an annuity in which the withdrawal equals the interest paid each time period and so the withdrawal can be made for ever.
Withdrawal will be $30 885.
Specific behaviours
✓ satisfactory explanation of perpetuity
✓ correct new withdrawal

Question 20   **(10 marks)**

A reverse mortgage is a loan that allows a person to borrow money using their home as security. Interest is charged like any other loan, but no repayments are made - the interest compounds over time and is added to the loan balance.

At the start of January 2019, a borrower takes out a reverse mortgage for $155 000 at an interest rate of 6.54% per annum, compounded monthly.

(a)   State a simplified recurrence relation for T_n, the loan balance after n months.   (2 marks)

Solution
$1 + 6.54 \div 12 \div 100 = 1.00545$
$T_{n+1} = 1.00545T_n, \qquad T_0 = 155000$
Specific behaviours
✓ interest multiplier
✓ recurrence relation

(b)   Determine the loan balance at the start of January 2022.   (2 marks)

Compound Interest

N	36
I%	6.54
PV	155000
PMT	0
FV	−188498.9443
P/Y	12
C/Y	12

Solution
Interest periods $= 3 \times 12 = 36$
$T_{36} = \$188\,498.94$
Specific behaviours
✓ number of periods
✓ correct balance to nearest cent

(c)   Determine the effective interest rate of the reverse mortgage, correct to 4 decimal places.
                                                                                        (1 mark)

Solution
$E = 6.7396\%$
Specific behaviours
✓ correct rate to 4dp

The borrower's home was valued at $420\,000$ at the time they took out the reverse mortgage and was expected to appreciate at a rate of 2.5% per annum.

(d)   Use a recurrence relation to determine the value of the home at the start of January 2022.

(2 marks)

| Solution |
|---|
| $V_{n+1} = 1.025V_n, \qquad V_0 = 420000$

 $V_3 = 452\,294.06$ |
| **Specific behaviours** |
| ✓ use of recurrence relation |
| ✓ correct value to the nearest cent |

(e)   At the start of which calendar year will the loan balance first exceed the value of the home? Justify your answer.   (3 marks)

Solution
After n **years**, loan balance: $B_{n+1} = 1.067396B_n, \ B_0 = 155000$ $B_{24} = 741565, V_{24} = 759665$ $B_{25} = 791544, V_{25} = 778657$ $2019 + 25 = 2044$ Loan first exceeds value at the start of year 2044.
Specific behaviours
✓ indicates method
✓ shows will take 25 years
✓ correct year

Question 21   **(10 marks)**

The number of people (n, in thousands) studying in Australia with a student visa from 2014 to 2016 are shown in the graph and table below.
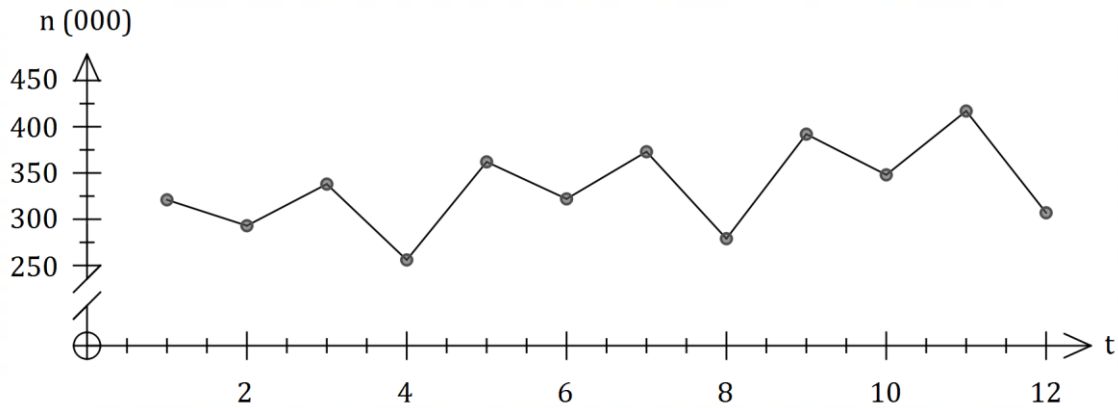
Year	Quarter	Time (t)	Number (n 000's)	Quarterly mean	Percentage of quarterly mean
2014	1	1	321		106.3
	2	2	293	A	97.0
	3	3	338		111.9
	4	4	256		84.8
2015	1	5	B		108.4
	2	6	322	334	96.4
	3	7	373		111.7
	4	8	279		83.5
2016	1	9	392		C
	2	10	348	366	95.1
	3	11	417		113.9
	4	12	307		83.9

(a)   Determine the value of A, the value of B and the value of C in the table above.   (3 marks)

Solution
$A = (321 + 293 + 338 + 256) \div 4 = 302$
$B = 4 \times 334 - 322 - 373 - 279 = 362$
$C = 392 \div 366 = 1.071 = 107.1$
Specific behaviours
✓ value of A
✓ value of B
✓ value of C
-1 if in thousands

(b) Complete the missing values in the seasonal index table below. (2 marks)

Quarter	1	2	3	4
Seasonal Index	1.072	0.962	**1.125**	**0.841**

Solution
$Q3: (111.9 + 111.7 + 113.9) \div 3 = 1.125$
$Q4: 4 - (1.072 - 0.962 - 1.125) = 0.841$
Specific behaviours
✓ one correct value
✓ both correct values adds 4 exactly

(c) Calculate the deseasonalised figure for the number of people studying in Australia with a student visa in the second quarter of 2016. (2 marks)

Solution
$348 \div 0.962 = 361.7$
Number of people is 361 700.
Specific behaviours
✓ indicates correct calculation
✓ correct figure, allowing for thousands

Let N represent all the deseasonalised values of n. The equation of the least-squares line for N against t is given by $N = 7.11t + 287.8$.

(d) Determine an estimate for the number of people studying in Australia with a student visa in the second quarter of 2018 if the existing trend and seasonality continues. (3 marks)

Solution
$t = 18$
$N = 7.11(18) + 287.8 = 415.78$
Estimate $= 415.78 \times 0.962 \approx 400$ thousand people
Specific behaviours
✓ uses $t = 18$
✓ correct value of N
✓ correct estimate, allowing for thousands

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____